Dynamic analysis of pedestrian bridges with FEM and CFD

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Summary

The paper describes a numerical study of dynamic response of a footbridge under pedestrian load and flow field created by big truck passing underneath. Standard FEM formulation is used to compute the response of the structure but the main problem discussed is how to define loads. Pedestrian load function is described in the first part of the paper. Lock in effect (interaction between pedestrians and the deck) for vertical vibrations is explained and the proposition of simple nonlinear dynamic model is presented.

1. Introduction

Modern footbridges are more and more sensitive to variable load created by moving pedestrians. There are several reasons for that. Quickly expanding road infrastructure, force us to build footbridges just to reconnect one piece of land separated now by the new road or railway. These bridges are delicate because the main live load acting on them comes from pedestrians. Very often there is no place for a pier and long spans are only feasible solutions. It happens often on the highway or in the city centers because of a visual aspects or collisions with underground facilities. Additionally new light and highly stressed materials are used for constructing. Finally lightweight structures are coupled with modern architecture and often armed with cable stays and suspension systems. In consequence mass and stiffness of structures decrease so much that the structures are sensitive to dynamic excitation.

The dynamic problems of footbridges have been subject of recent studies. Main reason of new engineering challenges related to footbridges is their low excitation energy. This makes them vulnerable to vibrations. Periodic load from pedestrians and wind can accelerate a bridge to the level which can be dangerous for structure itself or at least to large to be tolerated. Recent studies describe accelerations criteria admissible for user comfort, but often lack descriptions of loads. Therefore definition of loading is key aspect for proper implementation of these criteria.

2. Dynamic response under pedestrian action.

This part focuses on the definition of human induced dynamic load in the relation to body weight, pacing rate, density and interaction between pedestrian and vibrating deck (lock in effect).The response of a pedestrian bridge structure can be calculated with a commercial computer FEM programs if the dynamic load is defined. Bachman [1] has defined a pedestrian load function for *2 Hz* frequency. Kerr and Bishop [2] presented experimental results and conclusions for pace rates between *1.6* and *2.2 Hz*.

2.1. Load function from single pedestrian.

A special experiment has been done to develop more universal load functions. The strategy was to examine several humans walking with different pace frequency on the experimental set-up and to develop a load function.

The stiff steel double arm lever was constructed. First arm was connected to the digitally driven dynamic hydraulic jack. On the second arm a fitness treadmill with electric steering was placed. (Fig.1). Force cells were used as connectors between treadmill and a steel lever. Dynamic load induced by walking person can be measured and stored in data base. Experimental set-up allows additionally to measure loads on the vibrating deck. The interaction between walking person and vibrating structure can be also observed.

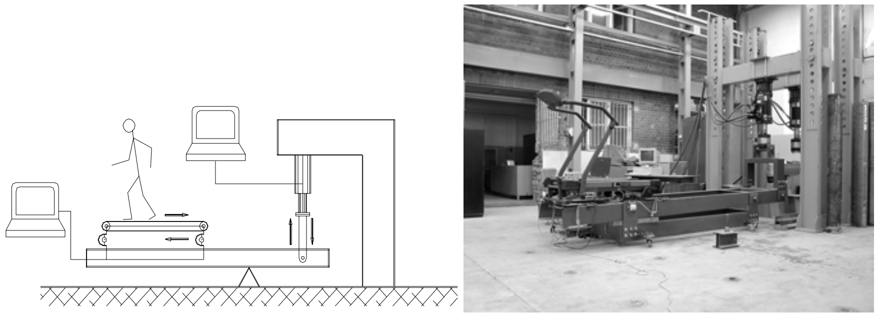


Fig.1. Fitness treadmill on the vibrating lever

A test was executed to find out a formula for human ground reaction forces during walking with different pace rate. 30 subjects were tested on the run way with pace frequency 1.3 Hz to 2.6 Hz. Basing on the test results a simple formula for human walking load was proposed (Eq. 1).

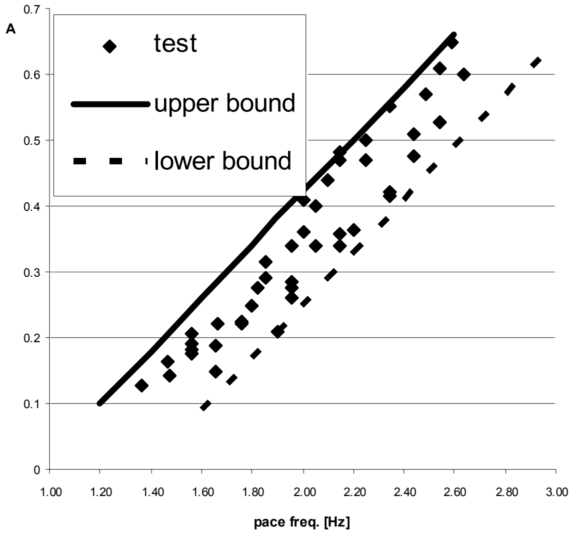


Fig.2 First harmonic amplitude of load function (Eq. 1)

For each single test, amplitude *A* has been calculated separately on the assumption given in Eq. 2. It means that function $F_h(t)$ has the same root mean square level “RMS” as original signal $q(t)$. This condition in effect creates equivalent (different from origin) load which gives the same response of bridge structure as origin. On fig. 2 calculated first harmonic amplitude *A* for $F(t)$ is shown. Basing on the presented result a simple formula for amplitude *A* was developed (Eq.3).

$$F_h(t) = BW\{1 + A[\sin(\omega_h t) + 0,25\sin(2\omega_h t + \pi) + 0,25\sin(3\omega_h t + \pi)]\} \qquad (1)$$

$$Z_s = P_s \quad Z_s = \sqrt{\frac{\int_o^T F^2(t)dt}{T}} \quad P_s = \sqrt{\frac{\int_{t_1}^{t_2} [q(t) - q_{av}]^2 dt}{t_2 - t_1}} \tag{2}$$

$$A = 0{,}4 \frac{\omega_h}{2\pi} + 0{,}6BW - 0{,}84 \tag{3}$$

BW – body weight [kN]
$q(t)$ – human induced load - experimental result
ω_h - pacing rate- result of Fourier analysis of test signal

2.2. Vertical lock in effect

“A walking person adapts to and synchronizes his/her motion in frequency and phase with vibrating deck. This phenomena depends from the human personal features and vibration characteristic of the deck “(Bachmann [1]). To develop a nature of this phenomena a laboratory test was executed. A nature of the human behavior during walking on the vibrating fitness treadmill was the purpose of the test. On the plot (Fig.3) history of vibrations of test platform (Fig.1) is presented. Amplitude of displacements is growing from zero to the constant value of 10 mm. The standard test was divided into four phases.

- Preliminary phase. The tested object was allowed in this phase to walk freely to the moment when he got used to walk on the fitness treadmill.
- Second phase (~10 s - first measured phase) it is the march on the stabile platform.
- Third phase (~10 s) it is the march on the platform. Amplitude of vibrations is growing linearly to determined value.
- Fourth phase (~20s) it is the march on vibrating platform with constant amplitude.

Since a dynamic load is the practical consequence of walk, variation of dynamic reaction under fitness treadmill was tested. History of dynamic pedestrian load is presented on Fig.4.

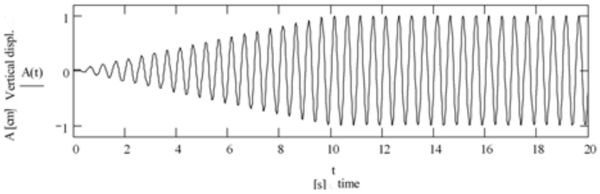


Fig. 3. History of deck vibrations

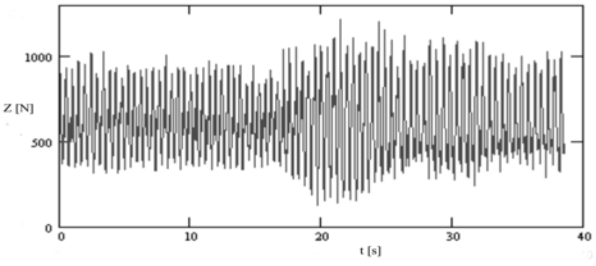


Fig.4. History of pedestrian load during test (time ~15s to ~25s is phase 3)

History of the load (Fig.4) is showing the reliable regularity. Pedestrian in the phase 3 (increasing of the vibration amplitude) is loading the platform more strongly than before (no vibration) and after (constant amplitude). Participants of the test determined the phase 3 as uncomfortable. The analysis of the movie shows, that pedestrian in the phase 4 is adapting the step to vibrations in order to reduce forces in legs and in consequence load action on the platform. Making the assumption that the dynamic effect of walking has the origin in the vertical movement of the body masses it is possible to define the simple hypothesis of adapting.

Vertical lock in effect during walk on the vibrating deck has an origin in natural adapting mechanism which reduces forces in human body. This effect can be reached if the canter of human mass (stomach) gets as small acceleration as possible.

The lock in effect is a synchronization of the frequency of steps and the frequency of the deck coupled with the phase shift. Analysis of test results and movies are pointing, that the moment of foot contact with the deck during walk occurs when the deck is moving down.

The lock in effect can be simulated by a simple kinematics formula in which phase shift is determined by minimum of RMS (root mean square level) from dynamic force acting on human legs.

Assumptions:

- pedestrian is represented as concentrated mass and one leg (the length of the leg is variable in time independently from deck), fig.5.
$$h(t) = H\,sin(\omega_h t + \varphi)$$
- deck is vibrating undependably form pedestrian
$$y_b(t) = U\,sin(\omega_b t)$$

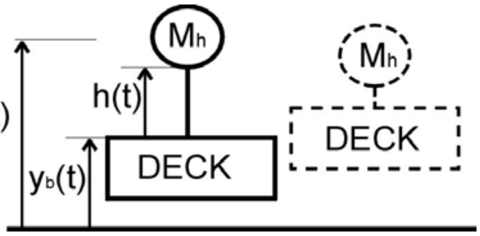


Fig.5. Pedestrian and vibrating deck – simple model.

M_h - human mass
$h(t)$ - distance from human mass to the deck (length of the leg)
$y_h(t)$ - vertical displacement of the center of human mass
ω_h - paceing rate of the pedestrian
ω_b - frequency of the deck

$$y_h(t) = h(t) + d(t)$$
$$y_h(t) = U\,sin(\omega_b t) + H\,sin(\omega_h t + \varphi) \tag{4}$$

Force in human leg:

$$P_h(t) = M_h y_h''(t) = -M_h\left(\omega_b^2 U\,sin(\omega_b t) + \omega_h^2 H\,sin(\omega_h t + \varphi)\right) \tag{5}$$

RMS (root mean square level) from dynamic force acting on human legs (Z_S)

$$Z_s(\varphi) = M_h\sqrt{\frac{\int_o^T y_h''(t,\varphi)^2\,dt}{T}} \tag{6}$$

Force in human leg is minimal if $:\dfrac{dZ_s(\varphi)}{d\varphi} = 0 \Rightarrow \varphi = \pi$

2.5. Single degree of freedom

The simplest model of SDOF system can simulate the phenomena of lock in effect (fig.6).

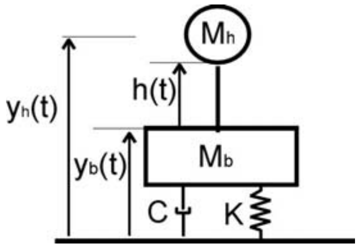


Fig.6. A simple model with a single degree of freedom

$$y_b'' M_b + C y_b' + K y_b + (h'' + y_b'') M_h = 0 \tag{7}$$

$$y_b'' M_b + C y_b' + K y_b + h'' M_h + y_b'' M_h = 0 \tag{8}$$

$$h'' M_h = F_h \qquad F_h(t) \text{ is defined in (1)}$$

$$y_b'' M_b + C y_b' + K y_b + F_h + y_b'' M_h = 0 \tag{9}$$

$$y_b'' M_b + C y_b' + K y_b + L_h = 0 \qquad L_h = F_h + y_b'' M_h \tag{10}$$

Equation (10) specifies a SDOF system with nonlinear external load $L_h(t)$. If the load and bridge frequency are equal and the load function is shifted by π in phase (to function of displacement of SDOF system- lock in effect) we can easily find out that $L_h(t)$ is decreasing in time to:

$$F_h(t) = A[0{,}25\,\sin(2\omega_h t + \pi) + 0{,}25\,\sin(3\omega_h t + \pi)] \text{ in case if } A\sin(\omega_h t) = -y_b'' M_h$$

This situation shows that a lock in effect can accelerate structure to the finite value. If we ignore damping in (10) and 2-nd and 3-rd harmonic in (1) the final acceleration of the deck with first natural frequency of 2 Hz (average pedestrian mass of 75 kg) is 6 m/s^2 and amplitude of deflection is 0,038 m.

2.2. Verification

The response of two footbridges has been measured under controlled pedestrian load to verify the proposed load function. First bridge is a steel continuous beam with orthotropic deck over Nogat river in Malbork and second is cable stayed steel bridge over Woloska street in Warsaw (fig.7)

Fig. 7. Bridge over Nogat river in Malbork and over Woloska street in Warsaw.

Bridge in Malbork has a total length 168,3 m divided to 7 spans (16,5 m, 2×24,0 m, 39,0 m, 2×24,0, 16,5 m). First natural frequency is 2 Hz. Bridge was loaded by 3 pedestrian walking with synchronized pace rate 1.6, 1.8, and 2 Hz. Footbridge over Woloska street is a pure steel structure. Main span is 62,9 m long. First natural frequency is 1.78 Hz. Bridge was loaded by 8 pedestrians walking with synchronized pace rate 2 Hz. Numerical models for both bridges were created with

SOFiSTiK FEM system. An interactive procedure of intelligent moving load (equation no. 10) was implemented to ASE module. Results of the test and simulation are presented on fig.8 and 9. Simulation with other (1.6 - 2.2 Hz) pace rate gives good compatibility too.

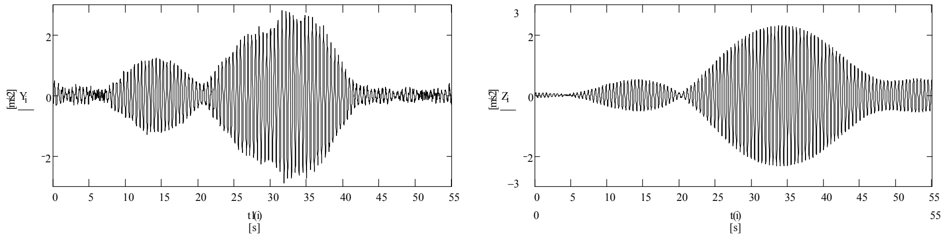


Fig.8. Malbork - max. acceleration of the main span. Test (left) and simulation (right)

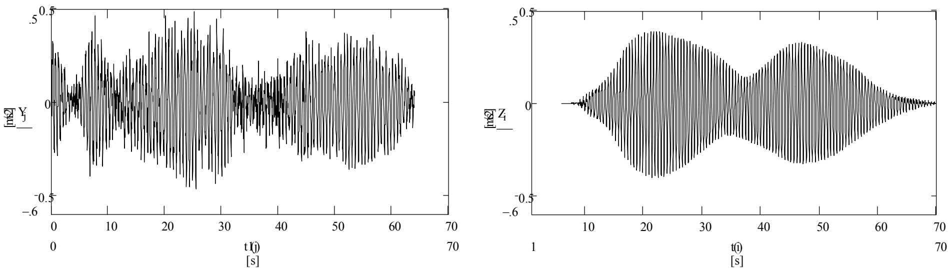


Fig.9. Woloska - max. acceleration of the main span. Test (left) and simulation (right)

3. Footbridge excitation from air impact of trucks passing underneath

3.1. Problem background

The dynamic action of truck's compression wave on lightweight footbridge has been noticed first (to our best knowledge) by Firth [3]. The problem was mentioned as a potential barrier on further lightening footbridges through introduction of lightweight materials. Following study of literature has shown that there was no interest in this action before. This kind of loading has been omitted as not significant for traditional heavy bridges mainly because it acts in opposite direction to dead load and also because there were no problems with this kind of action in he past.

Similar research, however not directly related to analyzed problem was performed by Shin – Park [4] and Fujii – Ogawa [5]. Their work was related to fast railway and problems of trains entering the tunnel or passing each other in the tunnel. Both teams used CFD for unsteady flow simulations. Both teams also used moving mesh technology and interfaces to simulate flow around objects moving against each other. Obtained results were verified by scaled-down testing. Concluding these works, forces on tunnel walls and train body are related to:

- tunnel geometry
- train body geometry
- train speed
- clearance between train body and tunnel walls

The procedure used for calculation of loading force on the footbridge span due to air impact from truck passing underneath resembles the approach of Shin – Park [4] and Fujii – Ogawa [5]. The following variables were chosen for the analysis:

- footbridge clearance – 4.7m which is minimum allowed for newly designed expressways in Poland,
- truck height – 4m which is maximum allowed by transportation administration,
- vehicle speed – 90km/h – maximum allowed speed for trucks in Poland,

The FEM model of footbridge over Wołoska street in Warsaw (Fig. 7, right photo) was selected for evaluation as the very good model verified by in situ testing was available.

3.2. Solution procedure

Simulations were performed using FLUENT 6.0 CFD code, based on finite volume method. Flow was treated as incompressible which is justified by small Mach numbers around 0.1. Therefore only momentum equations and continuity equations were solved. In order to account for turbulence a two equation $k - \varepsilon$ turbulence model was used. This model is based on turbulence kinetic energy k and its dissipation rate ε. The k equation is derived from exact solution, however ε equation is derived from averaged Reynolds stress tensor. Therefore molecular viscosity effects are omitted and fully turbulent flow is assumed.

Airflow around a vehicle moving at a constant speed can be assumed to be steady in ideal conditions. When the vehicle with its steady pressure disturbance zone moves in vicinity of another object, the pressure disturbance zone must deform and steady flow assumption is no longer valid. Steady object and moving body introduce new pressure disturbances, which can be interpreted also by means of forces acting on the bodies.

The problem of a vehicle moving under a footbridge is not a typical task for wind tunnel testing. This task requires a setup in which fluid (air) is not moving and the vehicle (truck) is moving. Typical wind tunnels allow simulations of moving air around not moving objects. They don't allow simulations of object moving relative to each other. The simplest way to setup testing for truck and footbridge is to use real footbridge and real truck.

CFD offers a solution to such simulation. Interfaces and sliding mesh technique were used to connect stationary zones with moving zone. Sliding mesh technique allows coupling of zones even if the nodes on two sides of the interfaces do not coincide. The equation system is modified dynamically with nodes changing their positions.

Computational domain was divided to three areas:

- stationary upper part with footbridge cross-section - "sky"
- moving mesh zone with truck shape - "lane"
- stationary lower part with road surface – "road"

Domain division scheme and used boundary conditions are presented on Fig. 10.

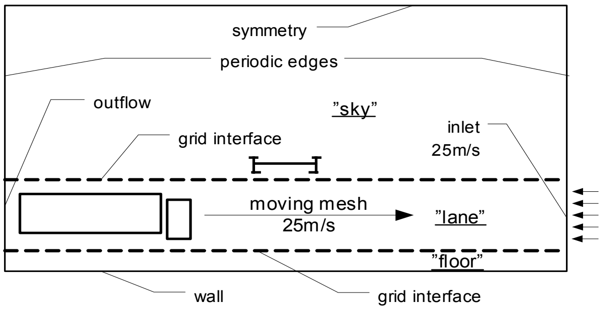


Fig. 10. Boundary conditions and CFD calculations setup

Truck geometry was simplified to very basic shapes. The truck with trailer and 40 ft. cargo container (Fig. 11) was used as a representative of common trucks found on Polish roads.

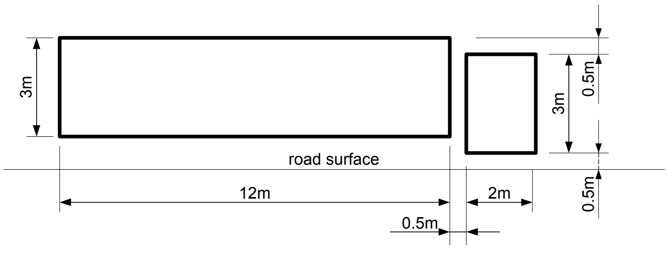


Fig. 11. Truck geometry

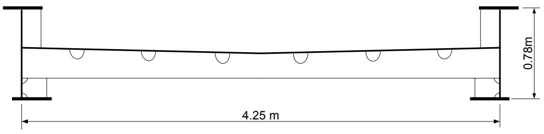


Fig. 12. Footbridge cross-section

The full 3D solution of the truck passage simulation was not possible due to limited computational power. Fine shape of footbridge cross-section required fine finite volume grid, which exceeded reasonable size. Therefore a 2D simulation was performed with exact shape of footbridge cross-section (Fig 12). Another 3D simulation was made with footbridge cross-section simplified to a bounding box. This 3D simulation along with a 2D simulation of a bounding box footbridge cross-section allowed to estimate of correlation coefficient between loading on the footbridge in a 2D and 3D case. While this assumption is quite rough, it allowed estimation of forces acting on footbridge on the basis of 2D simulation results. The forces on the deck obtained from 2D simulations are presented on Fig. 13. Flow velocities in the moment when truck approaches the footbridge are shown on Fig. 14

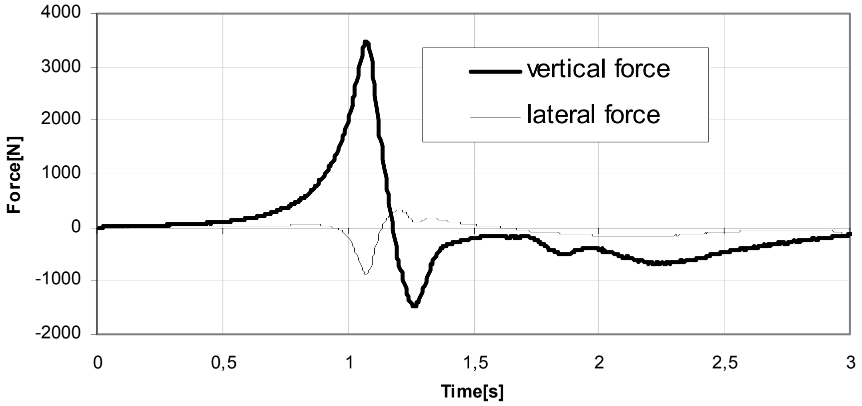
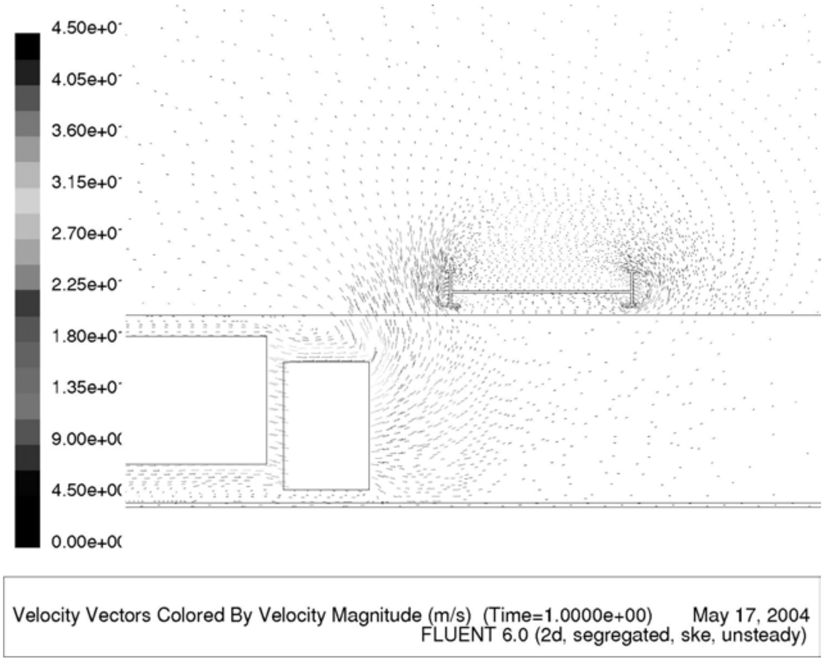


Fig.13. Results of basic 2D calculations (v=90km/h) with applied correction formula



Velocity Vectors Colored By Velocity Magnitude (m/s) (Time=1.0000e+00) May 17, 2004
FLUENT 6.0 (2d, segregated, ske, unsteady)

Fig. 14 Flow velocities when truck approaches the footbridge

The correction of 2D simulation results into pseudo-3D is done through a formula (11)

$$p_{3D}(t) = p_{2D}(t) * b * c \tag{11}$$

where:

$p_{2D}(t)$ - 2D model load function $[N/m]$

$p_{3D}(t)$ - approximate load function $[N]$

b - truck width, $b = 2.5[m]$

c - spatial distribution correction coefficient.

Value of c equal 0.55 has been found as appropriate for considered geometries.

3.3. Parametric study

Basing on the presented procedure, parametric calculations were performed. 2D models were used with key parameters influencing load functions changing with every calculation. All analyses were referenced to basic setup mentioned above. Parameters chosen for the analysis are structure clearance and vehicle speed. The results of the analyses are not multiplied by **c** scaling coefficient, because it applies only to the geometry used in 3D test. Changing structure clearance significantly changes spatial outflow and hence requires additional 3D calculations. These were very time consuming and 2D results are presented only for comparison purpose. Additional study was performed to account for the effect of a truck convoy. This was done to evaluate the risk of periodic excitation.

Raising the clearance of the structure seems to be an effective way of reducing forces acting on footbridge deck. Vertical loading functions for various structural clearances are shown on figure 15.

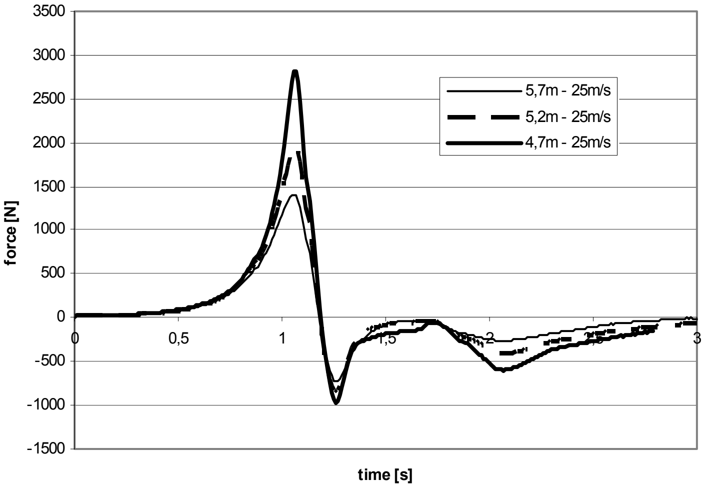


Fig. 15. Vertical load function with different structural clearances

Vehicle speed also has an effect on magnitude of loading. A truck traveling with excess speed of 126km/h (35m/s) causes twice as high peak load as the truck traveling at allowed speed of 90km/h (Fig.16).

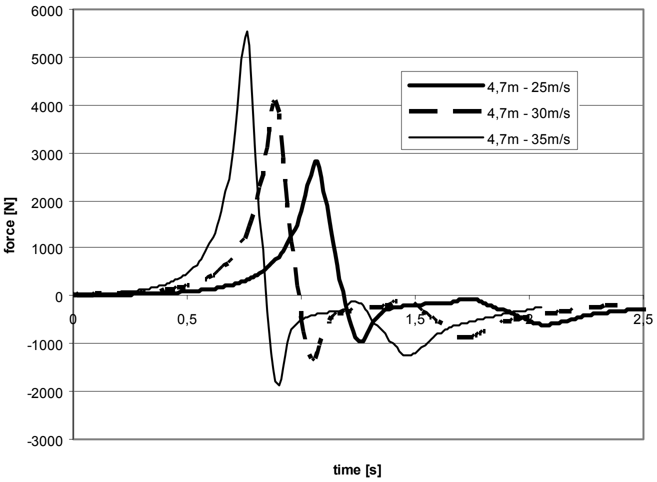


Fig. 16. Vertical load function with different vehicle speeds

A case with several trucks traveling one after another was also evaluated. Three convoy setups were studied with trucks spaced with 10m, 22.5m and 35m. In every case only the first truck caused impact same as single truck. The following trucks were traveling in aerodynamic wake and forces caused by them on the footbridge deck were significantly smaller. Loading function on the deck from a passage of four trucks spaced at 22.5m is presented on Fig. 17.

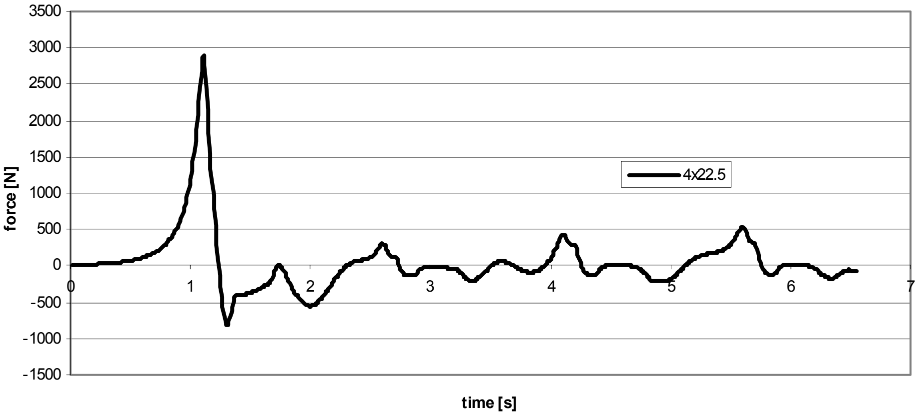


Fig. 17. Vertical load function from 4 truck with 22.5m spaces between trucks, v=90km/h

3.4. Structural dynamics analysis

The examination of dynamic response of footbridge over Woloska street was performed with use of FEM model of structure (Fig. 18) and SOFiSTiK DYNA module using direct integration of equations of motion. Beam elements were used to represent main structural members. The steel orthotropic deck was modeled with equivalent beam elements. The FEM model has been verified by field testing.

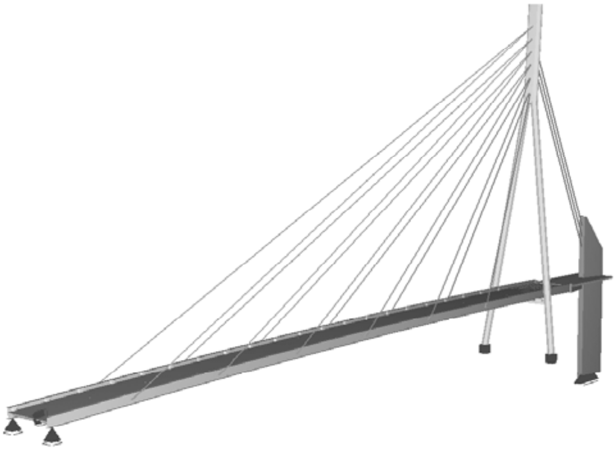


Fig. 18 Footbridge FEM model for transient dynamic simulations

The road under footbridge has two carriageways therefore it was critical to define right point to apply load from air impact. The most unfavorable case was when truck was traveling on the left lane of the carriageway further from pylon. Point on the main span coincident with the left lane was also the highest point of the first natural vibration bending mode of the footbridge. The CFD calculated loading components (Fy, Fz) were applied at this point.

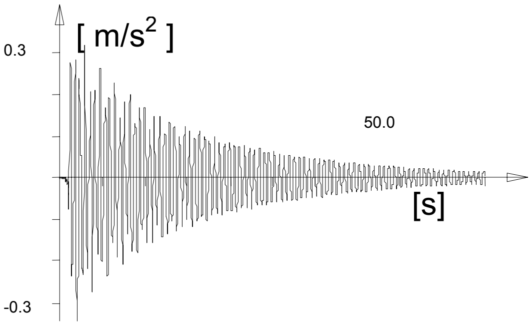


Fig.19. Point of maximum accelerations in deck . Response to load defined on Fig.14

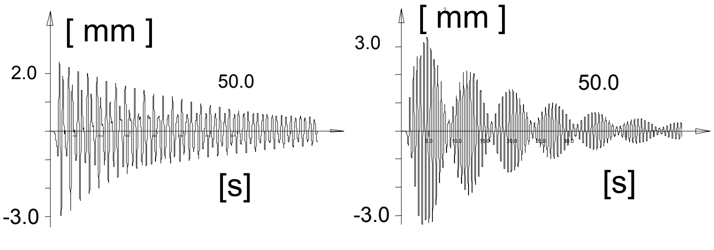


Fig.20. History of max. vertical displacement in stay nr 1 and nr 4.
Response to load defined on Fig.4

Time step of 0.005s was used. Total simulation time was set at 60s to allow free damped vibrations after truck has passed. The accelerations of the deck are presented on Fig 19 . Maximum vertical accelerations of 0.3m/s^2 were recorded during dynamic simulations. These values are acceptable in terms of pedestrian comfort, however they are noticeable by standing people. Another interesting result was obtained for stay cables. Fig. 20 shows displacements of the stay cables nr 1 and nr 4. These results point that the problem of steel fatigue in the stays due to repeating excitation might be worth checking.

While the evaluated footbridge is not the lightest one (steel orthotropic deck), observable amplitudes of vibrations were recorded in the simulation. This could have been much worse for a FRP-decked footbridge. Therefore air impact from trucks passing underneath could be a problem for a lightweight footbridge over a motorway.

4. Conclusions

Design procedures for modern footbridges are subject of continuous research. Much effort is spent on dynamics. The complexity of the problems is mainly related to couplings between varying loading and transient structure response. Therefore proper understanding of loading is necessary for a successful design. While researched procedures take time to spread into design practice, a flexible and powerful software tools reduce this time significantly.

Incorporation of intelligent loading to SOFiSTiK dynamics solvers and coupling them with CFD allows modeling multi-physics problems. Sample applications of these features have been presented in this paper. This unique interactive load – structure procedure allows us to estimate comfort properties of the structure – the next step after the function, structural strength and serviceability.

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